Dynamic Programming Workshop

Please ask our helpful programming team if you are confused or stuck! We are happy to help :)

1) **Fibonacci Sequence**

Find the nth number of the Fibonacci sequence. The Fibonacci sequence is a sequence whose first two elements are 0 and 1. Each following element is the sum of the previous two elements.

Constraints: $0 \leq n \leq 2 \times 10^5$.

Challenge: The new bound for n is $0 \leq n \leq 10^{19}$. You should find the nth Fibonacci number modulo $1\,000\,000\,007$ in a 1 second time limit.

2) **Maximum Sum**

You are given an array with n elements and want to choose some elements of the array such that sum of the chosen elements are maximum and no two adjacent elements are chosen.

Input Format

The first line contains an integer n ($1 \leq n \leq 10^5$) - the number of elements in the array.

The second line contains n integers $a_1, a_2, ..., a_n$ ($-10^5 \leq a_i \leq 10^5$).

3) **Minimizing coins**

Consider a money system consisting of n coins. Each coin has a positive integer value. Your task is to produce a sum of money x using the available coins in such a way that the number of coins is minimised.

For example, if the coins are $1, 5, 7$ and the desired sum is 11, an optimal solution is $5+5+1$ which requires 3 coins.

Input Format

The first input line has two integers n ($1 \leq n \leq 100$) and x ($1 \leq x \leq 10^6$): the number of coins and the desired sum of money.

The second line has n distinct integers $c_1, c_2, ..., c_n (1 \leq c_i \leq 10^6)$: the value of each coin.

4) **Coin combinations 1**

Consider a money system consisting of n coins. Each coin has a positive integer value. Your task is to calculate the number of distinct ways you can produce a money sum x using the available coins. In this problem, the order you choose the coins matter which means $2+2+5$ is different than $2+5+2$.

Input Format

The first input line has two integers n $(1 \leq n \leq 100)$ and x $(1 \leq x \leq 10^6)$: the number of coins and the desired sum of money.

The second line has n distinct integers $c_1, c_2, ..., c_n$ $(1 \leq c_i \leq 10^6)$: the value of each coin.

5) **Coin combinations 2**

Same as the previous question, but the only difference is the order you choose the coins don't matter which means $2+2+5$ is same as $2+5+2$.

6) **Elevator rides**

There are n people who want to get to the top of a building which has only one elevator. You know the weight of each person and the maximum allowed weight in the elevator. What is the minimum number of elevator rides?

Input Format

The first input line has two integers n $(1 \leq n \leq 20)$ and x $(1 \leq x \leq 10^9)$: the number of people and the maximum allowed weight in the elevator.

The second line has n integers $w_1, w_2, ..., w_n$ $(1 \leq w_i \leq 10^9)$: the weight of each person.