functional equations

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1 Problems

Find a function $f : \mathbb{R}^+ \to \mathbb{R}$ where $f(xy) = \frac{f(x)f(y)}{f(x)+f(y)}$
Find all functions which satisfy each property:

1. $f : \mathbb{R} \to \mathbb{R}, f(x+3) = x^2 - 3x$
2. $f : \mathbb{R}^+ \to \mathbb{R}^+, f(x) + 2f(\frac{1}{x}) = x$
3. $f : \mathbb{R} \setminus \{-1, 1\} \to \mathbb{R}, f(x)^2 f(\frac{1-x}{1+x}) = x$
4. $f : \mathbb{R} \to \mathbb{R}, f(x)y + f(x)f(y) = f(2f(x)f(y))$
5. $f : \mathbb{Z} \to \mathbb{R}, f(x+y) = f(x) + 2xy + f(y), f$ is continuous
6. $f : \mathbb{R} \to \mathbb{R}, f(x^2 + y) = f(x^{27} + 2y) + f(x^4)$
7. $f : \mathbb{R} \to \mathbb{R}, f$ is continuous, and $f(x^2) = xf(x)$
8. $f : \mathbb{R}^+ \to \mathbb{R}^+, f(f(x)) = 6x - f(x)$
9. $f : \mathbb{R} \to \mathbb{R}, f(f(x)^2 + f(y)) = xf(x) + y$