



Competitive  
Programming and  
Mathematics  
Society

# Mathematics Workshop #3

Problem Solving

**Cyril (with a side of Zac)**

# Table of contents



CPMSOC



## 1 Introduction

- Welcome

## 2 Tips and tricks

- Tips
- Experiment
- Draw a diagram
- Subproblems
- Reframing problem
- Reframing problem

## 3 Thanks for coming

# Welcome



- Next mathematics workshops in week 8
- Slides will be uploaded on website ([unswcpmsoc.com](http://unswcpmsoc.com))
- Pizza time soon™

# Tips

- Experiment
- Draw a diagram
- Subtasks
- Reframe the problem

# Experiment

- Try different values for numbers
- Try easier versions of the problem

# Experiment

A game is played between two players. Each player takes turns by removing anywhere from 1 to 4 stones from a pile. The person to remove the last stone wins. If both players play optimally, who will win if we start with 2023 stones?

# Experiment



A game is played between two players. Each player takes turns by removing anywhere from 1 to 2 stones from a pile. The person to remove the last stone wins. If both players play optimally, who will win if we start with 2023 stones?

# Draw a diagram

The integers 1 to 2023 are arranged in some sequence. One operation is made by swapping two numbers. What is the minimum integer  $k$  such that we can sort any sequence in less than  $k$  operations?



# Subproblems

Break the problem into steps which are, hopefully, solvable.



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# Subproblems

Find the maximum volume of a rectangular prism with a fixed surface area  $S$ .

- Is there a formula for the surface area and volume? Maybe  $2(ab + bc + ca)$  and  $abc$ .
- Is there a relationship between the two? Perhaps an *unequal* one?
- *Now* can we find the maximum?

# Reframing the problem

We've been doing this so far!





# Reframing the problem

There are 1001 points in the plane such that no three lie on a straight line.

The points are joined by 1001 line segments such that each point is an endpoint of exactly two of the line segments.

Prove that there does not exist a straight line in the plane that intersects each of the 1001 line segments (but not at their endpoints).

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This is problem A1 of the Simon Marais Mathematics Competition in 2020!

# Attendance code

