



PROBLEM SET 3

- 1 Choose some points in the plane such that no three of them form a triangle with area larger than one.
Show that these points all lie inside a triangle with area no larger than 4.
- 2 Let $b, r \in \mathbb{Z}^+$ and take $2r$ red points and $2b$ blue points in the plane, such that no three are collinear.
Show that there exists a line in the plane with exactly b points and r red points on each side.
Can you generalise this question to three dimensions?
- 3 Each point on the perimeter of an equilateral triangle is coloured either black or white.
Must there be some three points of the same colour that are also the vertices of a right-angled triangle?
- 4 Prove that any convex polygon of area one lies inside a rectangle of area 2.
- 5 Show that for all $n \geq 4$, there exists a convex hexagon which can be dissected into n congruent triangles.
- 6 For which n can you find n squares of different sizes that can be placed next to one another to form a rectangle? For example, does $n = 9$ work?