PROBLEM SET 1 - SELECTED RESOURCES AND SOLUTIONS

1 Positional analysis solution from StackExchange.

2 Search online for *Chomp*.

3 Search online for *Misére Noughts and Crosses*, and *3D Noughts and Crosses*.

4 Graph-theoretic solution from StackExchange.

5 Trivially, if the pile starts with an odd number of coins, first player wins by taking one coin. Therefore, the only non-losing move is to take an even number of coins such that the number of coins in a pile remain even. Suppose the pile starts with 2^n coins, where n $\in \mathbb{Z}^+$; the second player wins by simply copying the first player's move. If the game starts with any other number of coins, the first player can simply win by taking coins such that the remaining pile has 2^n coins.

6 Search online for *Crosscram*.

7 The operation performed on each pair is equivalent to addition modulo 2. Because this operation is commutative and associative, the final number is independent of the particular moves that are played, and so the winning player is determined at the beginning of the game: the first player wins if and only if there are an odd number of ones on the board.

9 Observe that if the resulting figure is ever a pentagon then the current player can move to form a hexagon so the next player loses. Therefore, we can assume that each player ensures that after their move, either a triangle or a paralellogram or trapezium is created. If we consider the side lengths of the parallelogram or trapezium then we observe that when each player moves, the pairs of side lengths decrease in "Euclidean algorithm style" where (m, n) becomes $(m-n, n)$. It is also possible for a smaller triangle to be created, so an inductive argument can be used. By using this information, it follows that the second player wins when n is a prime number or equal to 1.

10 Just because a question describes a game does not mean combinatorial game theory is the correct approach. This question is more about an invariant than a strategy.

(a) follows immediately from the pigeonhole principle, except in the $n = 10$ case. Divide the table into "even" and "odd" seats in an alternating fashion, with the girl the cards start with being in an "even" chair. Notice that the total number of cards in the even positions will remain even and likewise for the odd positions. Therefore it is impossible for there to be exactly one card in each position because then the total number of cards in even positions will be five, which is odd.

11 Partition the initial row of coins into "even" and "odd" sets. One of these sets begins with a monetary total which is greater than or equal to that of the other. By choosing either "even" or "odd" and only removing coins labelled as such, Ollie can guarantee a profit greater or equal to Ellie's.

13 Mirroring solution from MathOverflow.

14 Search online for analysis of *Hackenbush*.