Competitive Programming and Mathematics Society

Game Theory

Workshop 1, Week 3, Term 2, 2021

CPMSoc Mathematics

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What is a game?

Most games featured in competition problems are *combinatorial games*.

- There are two players who take turns to move.

Theorem (Fundamental Theorem of Combinatorial Games)

In any combinatorial game, there is a winning strategy for one of the players.

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Most games featured in competition problems are *combinatorial games*.

- There are two players who take turns to move.
- Both players have perfect information.
- The game ends after a finite number of moves.
- There are no draws [usually].

Theorem (Fundamental Theorem of Combinatorial Games)

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Position Analysis

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Theorem

In any combinatorial game, every position is either a winning position or a losing position. Also,

- *From any winning position, it is possible to move into a losing position.*
- *From any losing position, it is impossible to move into a winning position.*

Example: Impartial Rook

Strategy Analysis

A number of techniques can be used to analyse strategies as a whole.

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- Mirroring/pairing (e.g. two-pile Nim)
- Strategy stealing (e.g. tic-tac-toe)

Game Equivalence

A game tree is a representation of a game where nodes represent game states and directed edges represent valid actions.
Two games are said to be equivalent if there is a mapping between them that preserves the structure of the game tree.

Theorem (Sprague-Grundy Theorem)

Every finite impartial game under the normal play convention is equivalent to a one-heap game of Nim.